# Das Getriebesyntheseprogramm der ZG GmbH

# The Gear Synthesis Program of ZG GmbH

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# Kurzfassung

An derzeitige und zukünftige Getriebe werden zunehmend höhere Anforderungen gestellt. Neben größer werdender Gangzahl und der damit einhergehender größeren Spreizung stehen die Hybridisierung und der Getriebewirkungsgrad im Fokus.

Getriebestrukturen, mit denen die Anforderungen im Spannungsfeld aus Wirkungsgrad, Komfort (inkl. Akustik) und Kosten bestmöglich erfüllt werden können, sind aufgrund der Komplexität nicht mehr zielführend "per Hand" bestimmbar. Eine systematische und umfassende rechnergestützte Getriebesynthese ist notwendig. Bisher wurden Getriebesyntheseprogramme vorgestellt, die auf dem Prinzip beruhen, eine vorgegebene Anzahl an Getriebeelementen nach allen (bzw. sehr vielen) Möglichkeiten zu vernetzen und zu analysieren. Dabei sind als Getriebeelemente einerseits elementare Planetengetriebe (3wellig mit 2 Zentralrädern) bekannt sowie Getriebestufen als noch kleineren Einheiten daraus. Mit letzteren lassen sich auch direkt komplexe reduzierte Koppelsätze wie z.B. der Ravigneaux-Satz synthetisieren. Da die Getriebestrukturen aus ihren kleinsten Bausteinen aufgebaut werden, ist die zu berechnende Variantenzahl auch mit heutigen Großrechnern nur mit Einschränkungen praktikabel bewältigbar.

Grundsätzlich ist es günstiger, die kleinsten zu verschaltenden Elemente zu vergrößern um die Mächtigkeit des Variationsraums deutlich zu reduzieren. Dabei dürfen jedoch keine sinnvollen Lösungen übergangen werden. Die Vergrößerung der zu kombinierenden elementaren Elemente erfolgt im Syntheseprogramm "PlanGear", das eine Eigenentwicklung der ZG GmbH ist, auf Basis eines Analogieverfahrens. Für zahlreiche Getriebestrukturen lässt sich damit ein deutlicher Rechenzeitvorteil erreichen.

Im Rahmen des Vortrags wird der Grundaufbau des Syntheseprogramms dargestellt, das Analogieverfahren erläutert, und an einem ausgewählten Beispiel die Effizienz des Programms demonstriert.

#### Abstract

Present and future gears are facing higher and higher demands. In addition to the increasing number of gears and the accompanied development, which results in increased spread, hybridisation and gear efficiency grade are the main focal points.

As a result of their complexity, it is no longer productive to "manually" determine the gear structures to meet the demands which are best fulfilled from the conflicts between efficiency grade, comfort (incl. acoustics) and cost. A systematic and extensive computer-aided gear synthesis is required. The gear synthesis programs introduced so far have been based on the principle to link and analyse a given number of gear elements in all (respectively many) possible ways. On the one hand elementary planetary gears (3 shaft gears with 2 central wheels) are the known gear elements as well as gear unit stages are known as units even smaller. Even direct reduced complex compounded planetary gearsets such as, for instance, the Ravigneaux Gearset can be synthesised by means of these gear unit stages. Since gear structures are assembled from their smallest components, the number of variants can only be calculated in a limited way even by using modern high performance computers.

Basically, it is better to enlarge the smallest elements which are to be connected in order to significantly reduce the variants. However, no meaningful solutions must be ignored. The enlargement of the elementary elements to be combined is carried out by the "PlanGear" Gear Synthesis Program, an in-house development of the ZG GmbH, a program based on an analogue procedure. A considerably shorter calculating time can be achieved for a great number of gear structures by means of this program.

In the course of the lecture, the basic structure of the synthesis program will be presented, the analogue procedure will be explained and the efficiency of the program will be demonstrated with the help of a selected example.

### 1. Basic Information

In the past few years the ZG – Zahnräder und Getriebe GmbH has developed a Gear Synthesis Program which is able to identify complex gear structures (single and multiple gear structures). The algorithm used for finding the structure, however, is completely different from the procedures published as in [3] or [4]. In particular, when searching for multiple-coupled planetary gear structures and reduced complex compounded planetary gears of any type, it is possible to significantly reduce the search area thereby considerably reducing calculation time. Gear systems such as the active differential for Torque Vectoring Gears - as presented under [2] – can be found efficiently. The program resorts to the analogue procedure published by Helfer in 1967 in [1] which is briefly explained in the following.

# 1.1 Analogue Model according to Helfer

Helfer published an analogue procedure [1] basing on the similarity of a planetary gear and a lever. It can be demonstrated in a generally accepted way that torque moments and speeds at the shafts of a planetary gear are in the same ratio toward each other like forces and speeds in the nodal points of a lever. The nodal points represent the shafts of the planetary gear. The distances between the nodal points of the lever must have a specific ratio which is exclusively characterised by the transmission in the planetary gear.

**Figure 1** shows the analogue procedure by means of an example: If the sun shaft is determined in the planetary gear according to Figure 1, a fixed transmission ratio between carrier shaft and ring gear shaft will occur. The same applies to the lever: If the displayed lever is deviated around node 1, peripheral speeds will occur at the nodes; these speeds match the speeds in the planetary gear as long as distances a and b between the nodes are set correspondingly.

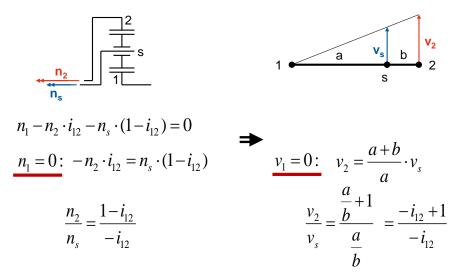


Figure 1: Rotational Speed- / Speed Analogy according to Helfer [1]

Also the torque forces developing at the shafts and the forces developing at the nodal points match proportionally if sections a and b have been set correctly. An example of this analogy is displayed in **Figure 2**.

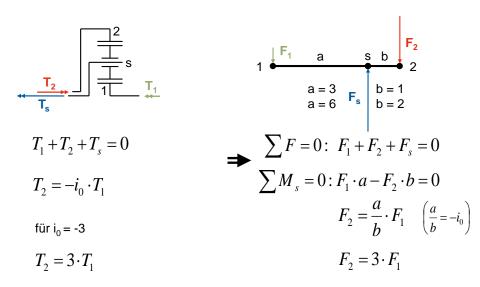
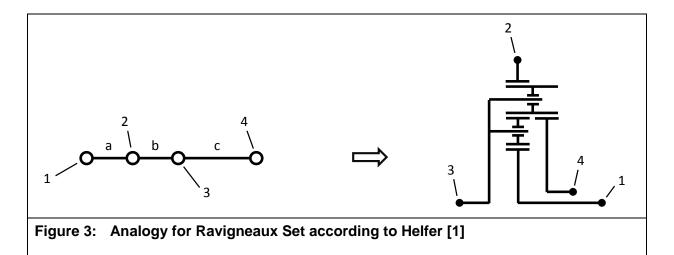


Figure 2: Torque- / Forces Analogy according to Helfer [1]

#### 1.2 Lever Model of a Ravigneaux Planetary Gearset

This analogue model does not only apply to single, elementary planetary gears but also to reduced planetary gears such as the widely-used Ravigneaux Planetary Gearset. The corresponding structure is displayed in **Figure 3**.



The lever from Figure 3 has been given section lengths in Figure 4 a); these section lengths enable the physical feasibility of the Ravigneaux Gearset. As known, the Ravigneaux Gearset, a reduced complex compounded planetary gear, consists of two individual elementary gears: for example, of a minus gear (central shafts running in opposite directions with a fixed carrier) and a plus planetary gear (central shafts running in the same direction with a fixed carrier). By means of the lever analogue model, it is possible to display this breakdown in a physically meaningful way. Figure 4 b) shows the breakdown in the aforementioned elementary gears. On the one hand we see the top lever, with now 3 nodal points, representing the plus gear consisting of the sun, 2 meshing planets, and the ring gear. On the other hand we see the bottom lever with 3 nodal points representing the minus gear consisting of two suns and two planets coupled with each other. Sun 1 and carrier 3 are part of both elementary gears and are connected thus. As Figure 4 c) shows, the connections in the lever analogue model match those levers to which the corresponding nodal points are coupled with. Thus, forces (corresponding to the torque forces occurring) and velocity (corresponding to the speeds occurring) are transmitted between the nodal points (correspond to the shafts).

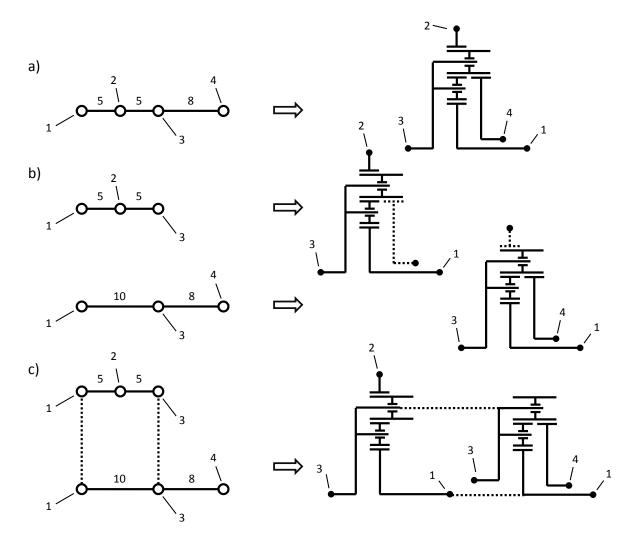


Figure 4: Breakdown of a Lever with 4 Nodal Points into 2 Levers each with 3 N. Points

Reduced complex compounded planetary gears are always equally effective with multiple coupled elementary gears. In an analogue model they can be depicted in the form of individual levers with the number of nodal points matching the number of shafts (incl. carrier shaft). The transmission ratios are determined by the section lengths of the levers (for example a, b and c in Figure 3). The decisive fact is that levers with more than 3 nodal points do not only represent a large number of reduced complex compounded planetary gears but also that they also represent all breakdowns in the respective elementary gears. In the case of the lever with 4 nodal points (in the following referred to as 4-lever) there is a total of 12 breakdowns into elementary levers with 3 nodal points (3-lever); the 6 mirror-reversed solutions, however, do not have to be taken into account. **Figure 5** displays all occurring breakdowns.

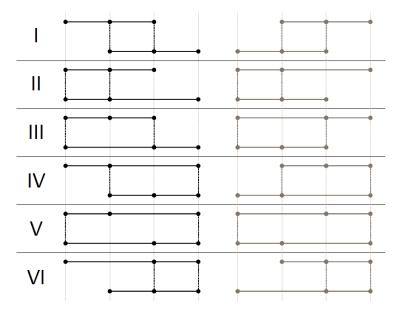


Figure 5: Possible Breakdown Options of the 4-lever into 3-levers (Elementary Levers)

By means of the lever analogue model it is possible to analyse and depict the entire statics and kinematics of a gearbox. In the following this will be explained by using the example of the ZF-9HP gear [5]. The structure of this gear is shown in **Figure 6** in the form of a stick diagram (converter not depicted). The gear is composed of 4 elementary planetary gear sets (minus gear) and 6 shifting devices with 2 jaw clutches and 4 power shifting elements being used. 3 of the shifting devices are brakes with which shafts can be halted in a manner fixed to the housing. Arrows symbolise drive and output. As can be seen in Figure 6, the two planetary gears close to output are referred to as compounded planetary gears II.

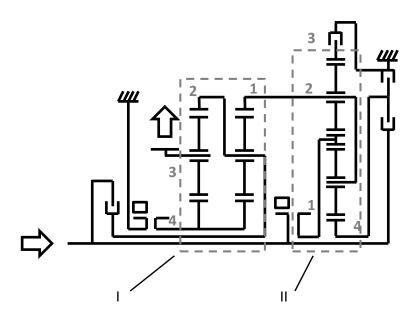


Figure 6: ZF 9-Speed Automatic Transmission (9HP) [5]

A closer look at the compounded planetary gear I shows that this gear is a double- coupled composite of minus gears, kinematically consisting of 4 shafts: shaft 1 serving as the ring gear of the minus gear positioned at the right side in **Figure 7** a), shaft 2 serving as the coupling shaft between the ring gear of the minus gear to the left and the carrier shaft of the gear to the right, shaft 3 serving as the carrier shaft of the left minus gear, and shaft 4 serving as the coupling shaft between both suns.

The analogue levers of the minus gear, too, can be coupled with each other by means of this structure. The 3- levers as depicted in Figure 7 b) show the corresponding coupling between the levers. The section lengths of the levers have been defined in an appropriate way so that the gear transmission ratios from [5] are achieved.

Finally, Figure 7 c) displays the merging of the levers to become one representative 4- ever.

An analogue presentation is the structure of the compounded planetary gear II which can be seen in **Figure 8**: Via shaft 1, the gear ring (with internal toothing) of the radially internal gear is coupled with the sun (with external toothing) of the radially external gear in a particularly favourable manner. It is thus possible to reduce the axial length of the gear compared to an inline arrangement. Shaft 2 couples both carriers, shafts 3 and 4 are composed of the remaining central shafts of the two minus gears. Initially, also compounded planetary gear II can be displayed via the two elementary levers (Figure 8 b). By means of the couplings given, this gear composite can also be displayed as 4-shaft-gear in the form of a 4-unit lever (Figure 8 c).

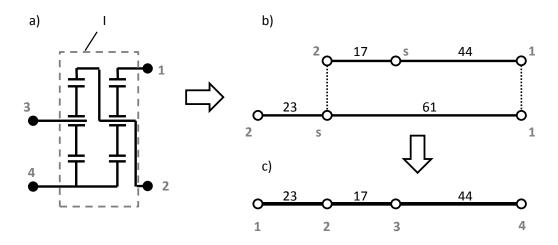


Figure 7: Compounded Planetary Gear I as Lever Analogue Model

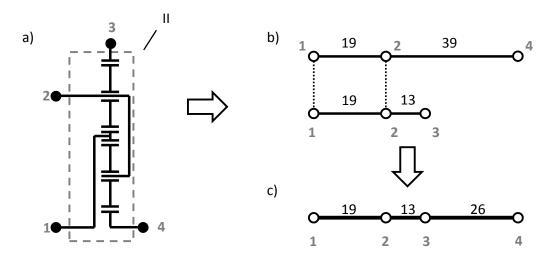


Figure 8: Compounded Planetary Gear II as Lever Analogue Model

**Figure 9** displays the structure of the levers for the first gear and the second gear of the gearbox. According to the leverage ratios it is possible to calculate the 1st gear transmission ratio of the gearbox by closing the grey marked shifting devices A, D and F and by resorting to **equation (1)**:

$$i_1 = \frac{(19+13)}{13} x \frac{(23+17+44)}{44} = 2,46x1,91 = 4,70$$
 (1)

$$i_2 = \frac{19+13+26}{13+26} x \frac{23+17+44}{44} = 1,49x1,91 = 2,84$$
 (2)

Exactly the same applies to the gear transmission ratio of the 2nd gear according to equation (2).

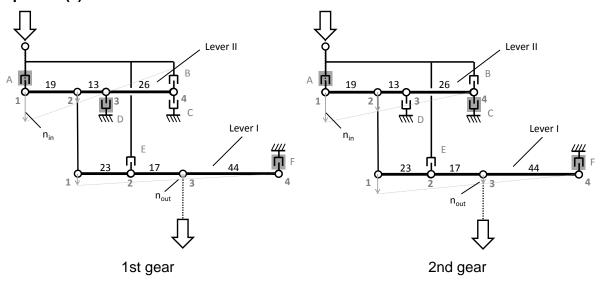


Figure 9: Structure of 1st and 2nd Gear

The interesting aspect about the structure of the 8th gear is the fact that the compounded planetary gear I is completely blocked due to the activation of both brakes (C and D). Therefore, shaft 1 of the compounded planetary gear I is halted due to the shaft bond (lever II, shaft  $2 \rightarrow$  lever I, shaft 1). According to **equation (3)**, the leverage which effects the gear transmission ratio can be determined by the lengths of the lever sections of lever I alone.

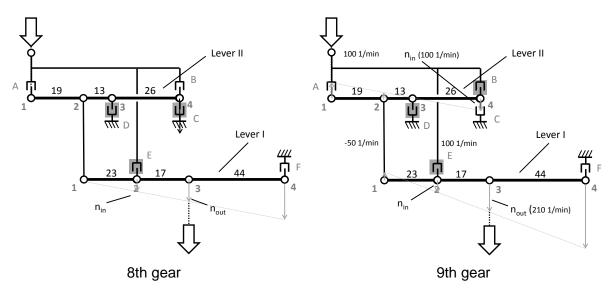


Figure 10: Structure of the 8th and 9th Gear

$$i_8 = \frac{23}{23 + 17} = 0,58 \tag{3}$$

$$n_{in} = 100 \tag{4}$$

$$n_{out} = -50 + \frac{150}{23}x(23 + 17) = -50 + 260 = 210$$
 (5)

$$i_9 = \frac{n_{in}}{n_{out}} = \frac{100}{210} = 0,48 \tag{6}$$

In the 9th gear, a speed ratio (corresponds to rotational speed ratio) which is created via lever II, is imposed on lever I. Via lever II, a speed direction (corresponds to the direction of rotation) in the opposite direction to the drive rate (corresponds to drive rotational speed) is created on node 1 of lever I. Via clutch E the drive rotational speed of the gearbox input shaft is directly transmitted to node 2 of lever I. Via the velocity difference and a given velocity, it is possible to draw conclusions regarding the speed of the output node (corresponds to output rotational speed) by resorting to **equations (4)** to **(6)**. According to equation **(6)** the ratio between drive velocity and output velocity corresponds to the transmission gear ratio of the 9th gear.

# 2. Gear Synthesis Program "PlanGear"

#### 2.1 Program Functionalities

In its current version (March 2014) the synthesis program PlanGear features the following key functionalities:

- Fully combined calculation of complex gearbox structures.
- Calculation of gearbox structures with 2 transmissions (hybrid transmissions), including the possibility to generate parallel and power-split structures.
- Very efficient synthesis of complex compounded planetary gears and reduced complex compounded planetary gears.
- Spur gear stages and / or planetary gears (any type) will be taken into account.
- Checking of assembly feasibility by means of graph theory (for any gear types).
- Calculation of operating conditions (torques, speeds, efficiency grade, etc.)

# 2.2 Gear Synthesis Process

The synthesis process is divided into two main steps by the PlanGear Program: During the first step the synthesis at lever level is carried out; during the second step the actual planetary gears are generated.

The key sequences of the gear synthesis program will be explained by using the example of the ZF-9HP Gear. The sequences can be divided into:

- 1) Specification of lever system
- 2) Specification of the basic conditions for synthesis at lever level
- 3) Synthesis process at lever level
- 4) Specification of the basic conditions for synthesis at gear level
- 5) Synthesis process at gear level

#### 2.2.1 Specification of lever system

To begin with, those levers have to be defined whose structures are to be synthesised. In the case of the ZF-9HP Gear there are two 4- levers and one shaft (see **Figure 11**). The specification of a 4-lever either implies a 4-shaft reduced complex compounded planetary gear (e.g. Ravigneaux or Wolfrom Set) or a double-coupled gear analogue to the structure as can be seen in Figure 7 or Figure 8. The type of the reduced complex compounded planetary gear or the structure of the double coupled gear composite has not yet been determined.

This synthesis task could also be solved by the specification of 4 elementary gears. However, the search for reduced structures would have to be carried out in the opposite way as depicted in Figure 4: All solutions resulting from two double-coupled elementary gears

would then have to be checked for reducibility - a process which requires calculating time and which is accompanied by the risk of overlooking specific reduced complex compounded planetary gears. The significantly higher calculating time for the synthesis of elementary gears is, however, far less advantageous. Indeed, calculation with elementary gears connected in any form provides a larger set of solutions because also those solutions will be detected which do not have double-coupled structures.

Also the synthesis of elementary gears (specification of 3- levers) is made possible by the PlanGear Program. The specification of compounded structures i.e. 4- levers, 5- levers or even 6- levers, however, significantly reduces calculating time and may result in very good gears as well. Thus, the number of structures checked per time unit can be maximised. In addition to this, also more complex structures can be calculated, structures such as the 14-speed Rohloff Hub Gear (non-shiftable under load) which consists of one 3- lever and two 4-levers, thus corresponding to five elementary gears. Even modern large-capacity computers would reach their limits without the specification of further basic conditions.

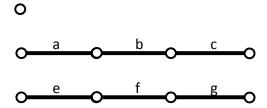


Figure 11: Lever Specification for the Synthesis of the ZF-9HP Transmission

#### 2.2.2 Specification of the Basic Conditions for Synthesis at Lever Level

Synthesis at lever level requires further basic conditions. Among these we find:

- Specification of gear ratios or of gear steps
- Maximum number of shifting devices
- Number of shifting devices which are closed when changing gears
- Number of maximum open shifting devices
- Maximum number of clutches (rest of shifting devices consists of brakes)

The higher the number of basic conditions specified for the synthesis process, the smaller the scope for solutions and the shorter the respective calculating time of the synthesis will be.

# 2.2.3 Synthesis Process at Lever Level

The synthesis process copes with all possible structures of the lever system and looks for those solutions by means of which the specified gear ratios or steps can be displayed. In order to modify the transmission ratios, the lever section lengths too will be varied during this process. The part of the section lengths to be examined can either be preset directly or can be determined in a meaningful way for specific planetary gears (e.g. Ravigneaux Set) by the PlanGear Program.

The synthesis process is a process parallelised via multithreading so that the calculating capacity available can be used in an efficient manner. At present the ZG GmbH resorts to a server with 64 logical processors (hyperthreading) and 384 GB RAM. The large storage enables rapid access to (almost) all synthesis data during the synthesis process without having to resort to time-consuming hard drive access (or establishing an expensive database management system). Frequently occurring computer operations such as solving the system of equations or checking the assembly feasibility by means of the graph theory are consequently converted into C or Fortran and stored in DLLs.

The key results of the gear synthesis at lever level are as follows:

- Section lengths of lever system
- Gear-related structure of levers
- Shaft operating data of gear (nodal points of levers): torque, revolution speed, power
- Switching matrix
- Torque forces relating to the initial conditions as well as rotational speed differences in shifting devices

The output of the gear synthesis for the first gears of the 9HP Transmission is exemplified and depicted. Compared to Figure 9 the graph automatically produced by the PlanGear Program (Figure 12) shows the same structure. Permanent shaft connections between the nodes (which correspond to planetary gear shafts) are displayed as blue dashed lines; shiftable connections (clutches) are displayed as orange dashed lines. Housing connections (which are not in the graph) would be nodes filled with red; brakes are displayed in the form of nodes filled with orange. Thus, even complex gearbox structures can be depicted in a clear manner so that they can be followed in a relatively easy way. In particular, the fact that shaft speeds (blue numbers) are displayed makes it easier to get an idea about the kinematics of the gearbox. Due to the indicated shaft power (normally shown in the colour grey) it is easy to detect the occurrence of any flow of idle power. Should any flows of shaft power occur which are bigger than the drive power, these flows will be marked with the colour red.

The shifting matrix of the gear is shown by **Figure 13**. In addition to the gear ratios and the gear steps, we can see the active shifting devices which have been marked by an "X". The red colour represents a closed brake, the green colour a shifted clutch.

The related operational demands, as can be seen in **Figure 14**, are displayed in the form of a matrix which has the same structure as the shifting matrix. The markings of the condition by means of an "X" in active shifting devices have been replaced by drive torque-related torque forces – in the case of open shifting devices, the drive speed-related difference in speed of rotation is displayed in the shifting device.

All results will also be exported to a csv file so that easy post-processing for all solutions found is possible.

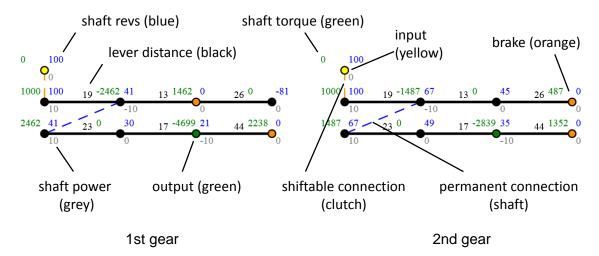


Figure 12: PlanGear Output: Lever Structure

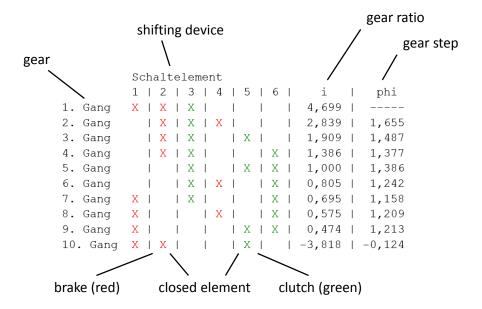


Figure 13: PlanGear Output: Shifting Matrix

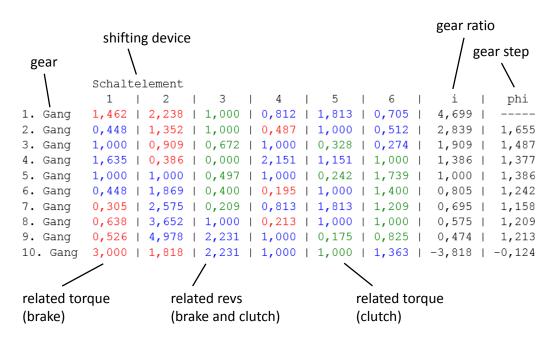
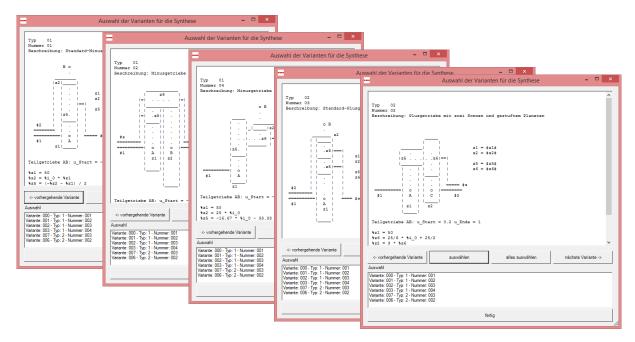


Figure 14: PlanGear Output: Shifting Matrix with Related Operational Demands

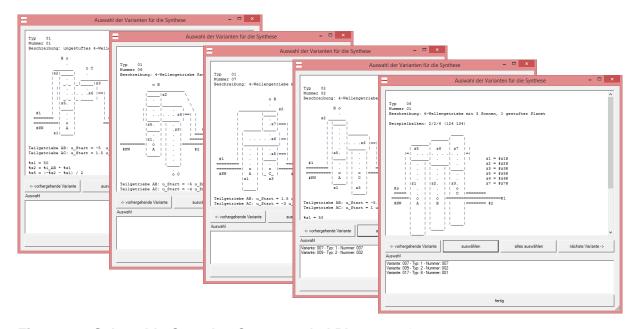
#### 2.2.4 Specification of the Basic Conditions for Synthesis at Gear Level

If all possible lever systems have been determined after the first synthesis step, it is possible to synthesise those planetary gear systems which are structurally and kinematically equivalent to the lever systems.

For this purpose, the PlanGear Program will ask the user for the construction type of those planetary gears which are to be looked for in the lever systems. As can be seen in **Figure 15** these types might be elementary minus gears and plus gears of various construction types. It is also possible to select from a catalogue of reduced planetary gears (see **Figure 16**); the number of the shafts of the reduced planetary gear must match the number of lever nodes.



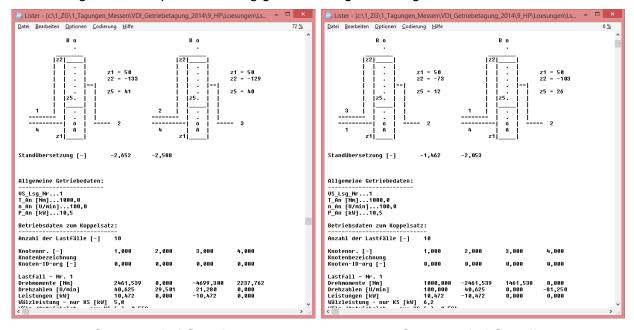
**Figure 15: Selectable Basic Planetary Gears** 



**Figure 16: Selectable Complex Compounded Planetary Gears** 

# 2.2.5 Synthesis Process at Gear Level

After the selection of those planetary gears which are to be looked for in the lever systems, the PlanGear Program will synthesise those planetary gears for all lever systems which have been determined during the first synthesis step. Via the analogue model, each lever of the corresponding gear system will be transferred to one or several planetary gears. In case of a split into elementary minus gears, the two 4- levers of the 9HP Transmission will result in compounded gears (and others) as displayed in **Figure 17**; the structure of these compounded gears is the same structure as in the 9HP Transmission. The number representing the connecting shaft in the diagram is equivalent to the number of nodes and thus to the number of lever shafts. The same figures represent coupled shafts. On the left side of Figure 17 (compounded gear I), for instance, the carrier shaft of the left minus gear is connected to the ring gear shaft of the right minus gear. The same applies to both sun shafts. In compounded gear II, on the right side of Figure 17, both carrier shafts and the sun of the left minus gear are coupled to the ring gear of the right minus gear.



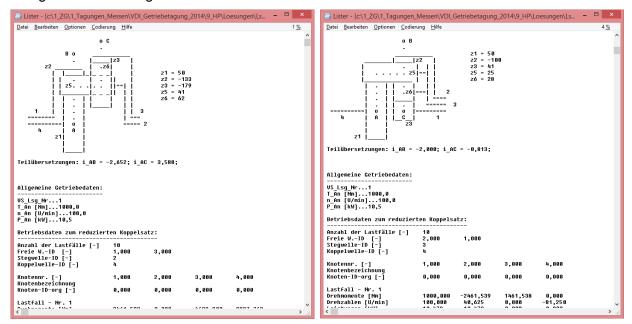
Compounded Gear I

Compounded Gear II

Figure 17: PlanGear Output: Compounded Gear Sets

Figure 17 does not display the entire output. The PlanGear Program determines and outputs all torque moments, all speeds and all power-flow in the gearboxes. Via the determination of the teeth number ratios, the calculation of the planetary bearing speeds is carried out, too. The toothing efficiency grade of the gearbox for all gears is determined by means of the mesh power flows in the individual elementary gearboxes and a gear toothing efficiency grade.

Further solutions, in the form of reduced complex compounded gears, can be derived from the results of the first synthesis step. An example of such a solution is depicted in **Figure 18**. A reduced gear with one sun and two ring gears is selected for lever I which corresponds to complex compounded gear I. The figures in the gear diagram in turn correspond to the node of the lever. A Ravigneaux Set is selected for complex compounded gear II, as can be seen in Figure 18 on the right hand side.



Complex compounded Gear I

Complex compounded Gear II

Figure 18: PlanGear-Output: Complex compounded Gear Sets

All torque moments, speeds and power-flows (incl. mesh power flows) will also be determined for reduced complex compounded gear sets. Also here, the calculation of the planet bearing speeds is enabled by determining the teeth number ratios.

**Figure 19** shows the gear set which is statically and kinematically identical to the 9HP gearbox. There are only two running carrier shafts left because reduced complex compounded gears generally have just one single carrier. The basic structure of the shafts is identical to the 9HP Transmission because it is the same lever analogue model which forms the basis. The torque moments in the closed shifting devices as well as the rotational speed differences in the opened shifting devices are thus also identical.

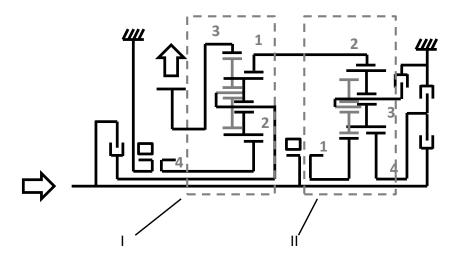


Figure 19: Alternative Gear Structure with Identical Ratios and Different Gear Sets

However, different mesh power flows occurring within complex compounded planetary gears cause a change of the efficiency grade as can be seen in **Figure 20**. Except for the 5th gear, where all shafts rotate at the same speed (during coupling), the toothing efficiency grades differ from each other. While the structure of the 9HP Transmission results in a more favourable efficiency grade in the lower gears, the alternative gear structure is the more advantageous structure in the gears 8 and 9. As already mentioned, the losses occurring in the open shifting devices continue to be theoretically constant compared to the 9HP Transmission. This is because the differences in speed of rotation are identical. However, it is very likely that the new basic constructional conditions would result in a change in the shifting devices (e.g. diameter deviation), a change which would then result in a change in the drag losses at the same differences in speed of rotation.

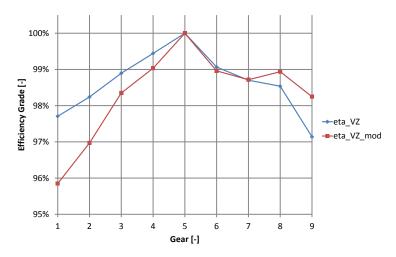


Figure 20: Toothing Efficiency Grade, Comparison

#### 3. Summary

The ZG – Zahnräder und Getriebe GmbH (Limited Liability Company) has developed the "PlanGear" Gear Synthesis Program in order to enable the determination of complex single and multiple gear sets. The PlanGear Program enables a two-part synthesis process. During the first step the kinematics and the statics of the gear set will be determined via a lever analogue model. At this point, the structural design of the planetary gear set is still irrelevant. Due to the fact that the determined levers represent an analogy to a large number of planetary gears and complex compounded planetary gears, the number of solutions for the synthesis process will be relatively small during the first step and therefore the calculating time will be short. By means of the lever systems defined for the solutions, however, it is already possible to determine the essential features of the gears and to make an appropriate selection.

Analogue gearbox systems for the lever systems will not be synthesised until the second step; however, this will only be done for the solutions selected. Planetary gears will be selected from a catalogue. These planetary gears will be checked with regard to their lever analogy. The outcome of the second step will then be the actual gearbox structures with all respective operating data. This may be 3-shaft elementary planetary gears as well as any kind of reduced complex compounded planetary gears.

In the course of the report, the basic procedure of the PlanGear Program gear synthesis was demonstrated using the example of the ZF 9HP Transmission. Using a current desktop computer model with 8 logical processors, the duration of the synthesis process for the ZF 9HP Transmission was approximately 1 day. One feature which has to be particularly highlighted is the efficient identification of the structures for reduced complex compounded planetary gears. In addition to constructional benefits due to the reduced number of rotating carriers, reduced planetary gear sets may result in reduced gear toothing losses due to the special design type of planetary gear sets making the flow of idle power within a planetary gear set impossible.

#### 4. References

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