An easy-to-use and fast computational model for the prediction of the influence of manufacturing errors on gear transmission error

Conference Proceedings

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Abstract

This paper presents an efficient tool for analyzing the transmission error (TE) in spur gears and helical gears under low load. The TE, i.e , a fluctuation of the transmission ratio along the line of action, is a result of tooth deformation, mesh misalignment, and deviations from the ideal gear geometry due to manufacturing tolerances. The unsteady motion transfer between meshing gears affected by TE is regarded as the main source of excitation for gear whine. Typically, the microgeometry of the tooth flank is modified in order to compensate for tooth deflections and mesh misalignment under high loading conditions. In low load operating scenarios, however, the deformations are negligible compared to the magnitude of manufacturing errors which may then cause severe NVH issues even though they lie within the requested tolerances.

By several parametric studies we demonstrate the influence of deviations from the ideal gear geometry on the TE and its frequency spectrum, considering: (i) profile barreling; (ii) lead crowning; (iii) pressure angle deviation; (iv) helix angle deviation; (v) profile waviness; (vi) bias; and (vii) pitch error. This study provides valuable insights into how manufacturing errors may enrich the excitation spectrum, e.g., by dominant sidebands and a-priori unexpected low frequency excitations. Furthermore, causal links between NVH behavior and relations between micro- and macrogeometry parameters are revealed. Such correlations allow to predict the sensitivity of gear designs to manufacturing errors and may aid in choosing a suitable set of flank modifications.

The presented model approach can be employed as a simple tool to gain understanding of fundamental interdependencies between gear quality and TE. It could aid in quality management at the end of the production line in order to assess the severity of deviations from the desired microgeometry and in troubleshooting noticeable NVH problems.

1. Introduction

In theory, a perfectly manufactured and rigid gear pair would have no transmission error (TE) because the transmission ratio would be constant along the whole line of action (LOA). In reality, however, the gear ratio fluctuates as the contact points between pinion and wheel move along the LOA. The TE is defined as the deviation of the actual pinion position on the LOA from the ideal position, and it originates from: (i) deformation of the meshing teeth; (ii) mesh misalignment; and (iii) manufacturing errors. Figure 1 illustrates the TE resulting from a geometric imperfection of the tooth profile. Fluctuations in the mesh force induced by the TE excite structure-borne noise which is transmitted via gears, shafts, and bearings to the housing from where it is emitted as airborne sound to the environment. Thus, the TE is regarded as the main source of excitation for gear whine.

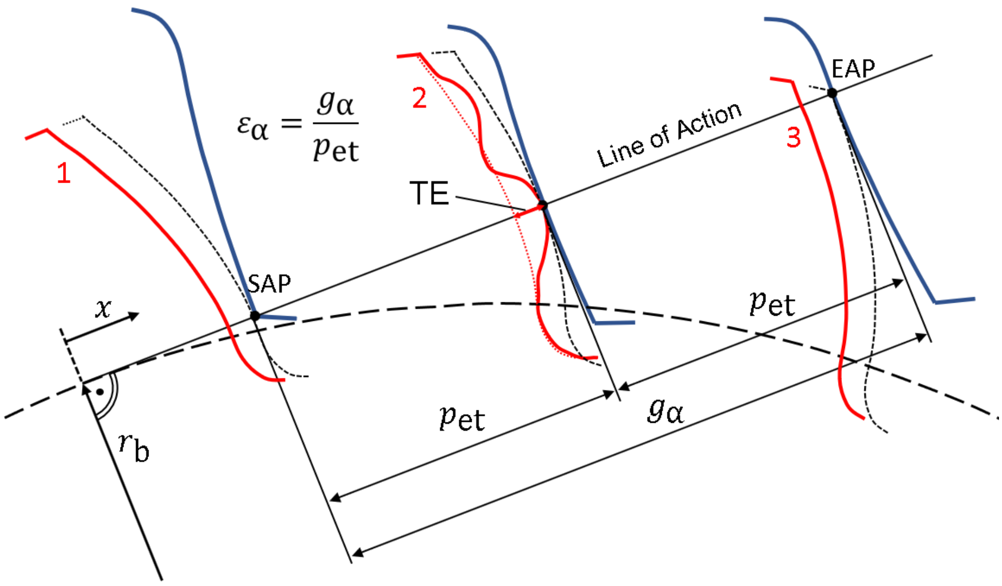


Figure 1: Transverse section with line of action (LOA) and 3 tooth pairs which are theoretically in contact. However, pinion tooth 2 has a flawed profile and is in contact with the opposing wheel flank at a peak with excess material compared to the ideal profile. Thus, teeth 1 & 3 are pushed away from their ideal position on the LOA.

If the load on the meshing gears is high enough the deformation due to tooth bending, the flattening of the contact zones, as well as the mesh misalignment due to shaft deformation and bearing deflections are usually dominating the TE. At moderate loads, however, the gear manufacturing errors and bearing mounting errors are the main source for TE. In order to minimize the TE and to achieve a quiet operating behavior, the common practice in gear design is to optimize microgeometry modifications on the gear flanks to compensate for the system deformation at high loads and to accept a compromise for low load scenarios. In the automotive industry time and cost pressure make it hard to achieve ISO quality grades below 5 in series production. At achievable quality grades the tolerance bands are so broad that the effect of manufacturing errors on TE and consequently on NVH are still prevalent in the medium load range. When facing NVH issues associated with low loads many engineers rightly suspect the gear quality as root cause but fail later on to identify the link between the measured gear topologies to the order cuts from the NVH measurements, either due to the lack of suitable analysis tools or due to widespread misconceptions on the influence of flank form deviations on the TE and its amplitude spectrum.

In this paper we address some of these common misconceptions and we reveal some fundamental but nonetheless valuable dependencies of the TE on the micro- and macrogeometry. The efficient and lean computational model presented in this paper can be a valuable alternative whenever noise issues in moderate load ranges need to be correlated with quality measurement protocols, or whenever quick statements on the severity of specific quality issues need to be made at the production line, or even in the design phase in predicting the susceptibility of gear designs to manufacturing errors.

2. Model for TE calculation

The conceptual idea behind our modeling approach exploits that the mesh of any arbitrary pair of gears can be represented by the line of action (LOA), the start of active profile (SAP), the end of active profile (EAP), the transverse base pitch p_{et}, the helix angle on the base circle β_{b}, and the gear width b. The LOA starts at roll distance x_0 tangent to the base cylinder of the pinion. The points of SAP end EAP are denoted x_{SAP} and x_{EAP}, respectively. If the LOA is swept along the gear width coordinate y the plane of action (POA) is generated. In helical gears the contact points associated with each tooth i in any transverse section are shifted on the LOA by $\Delta x_{\beta}(y, \beta_{\mathrm{b}})$ depending on the helix angle generating the contact lines $\boldsymbol{x}_i(y, \beta_{\mathrm{b}}, t)$. Since the contact lines move with pinion rotation, their position depends also on the time t. Note that only those fractions of the contact lines $\boldsymbol{x}_i^{\mathrm{A}}(y, \beta_{\mathrm{b}}, t)$ are active which lie between x_{SAP} and x_{EAP}. The microgeometry can either be generated from measurement protocols or designed within the program as composition of various modifications (see Figure 2). The microgeometry is mapped onto the POA and the deviation from the ideal flank is measured along the z-axis. As the pinion drives the wheel the contact lines roll over the hills and valleys of the microtopology.

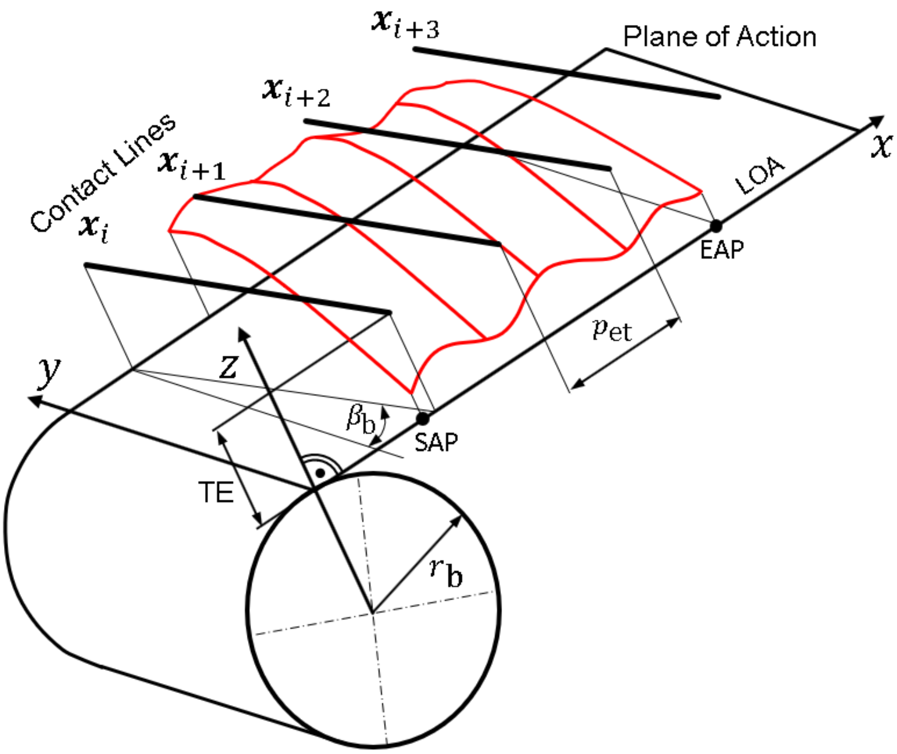


Figure 2: The plane of action (POA) is tangent to the base cylinder with radius r_b. The helical teeth are inclined at the base helix angle β_b. The flank of any tooth i intersects the POA in the contact lines $\boldsymbol{x}_i$.

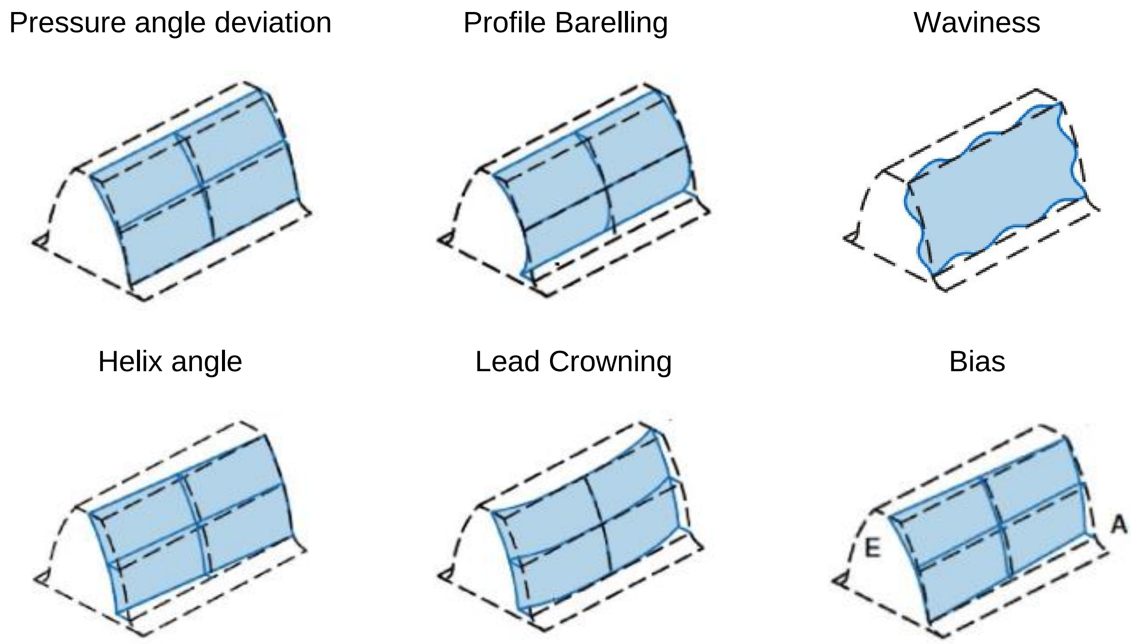


Figure 3: Illustration of microgeometry modifications [1].

Because in this model the contact lines are constrained to stay parallel and are not allowed to rotate around the LOA, they are always forced to lie on a plane parallel to the POA at a height z which is given by the highest point of the microtopology touched by the active fraction of any contact line. This process is illustrated in Figure 3. Neglecting deformation this is per definition the value of the TE:

$$TE(t) = \max_{x \in \mathbf{x}_i^{\mathrm{A}}} [z(x,t)] \tag{1}$$

3. Common Misconceptions

Misconception 1 – TE due to profile waviness

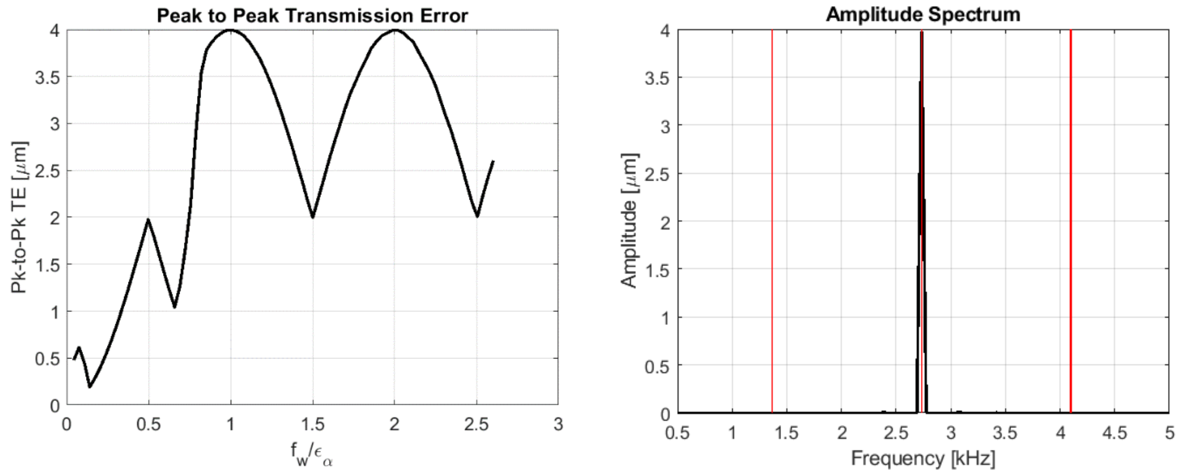
A common misconception is that the number of peaks along the tooth profile of a waviness on the flank correlates directly with the multiple, i.e., the harmonic, of the gear mesh frequency (GMF) that is excited. Accordingly, two peaks along the profile are believed to excite the 2nd harmonic of the GMF, three peaks excite the 3rd harmonic, and so on. In order to investigate the influence of profile waviness systematically, we start by defining a spur gear with parameters as given in Table 1. We define a sinusoidal profile deviation $s(x)$ expressed in the LOA coordinate x as follows:

$$s(x) = A_{\mathrm{W}} \sin\left(2\pi f_{\mathrm{W}} \frac{x - x_{\mathrm{SAP}}}{g_{\alpha}}\right), \tag{2}$$

wherein A_{W} is the amplitude of the waviness and f_{W} is the frequency. The length of the path of contact between SAP and EAP is denoted by g_{α} (see Figure 1). So $f_{\mathrm{W}} = 1$ means that there is one sine between SAP and EAP. The same profile deviation $s(x)$ is prescribed at all teeth of the pinion and the wheel is assumed to be ideal. Figure 4 shows how the peak to peak TE (PPTE) develops with increasing $f_{\mathrm{W}} \in \{0.1, \dots, 6.0\}$. The amplitude was fixed to $A_{\mathrm{W}} = 2\ \mu\mathrm{m}$. It is obvious that there are two maxima at whole-numbered values of the ratio between the frequency of the waviness and the transverse contact ratio $f_{\mathrm{W}}/\varepsilon_{\alpha}$.

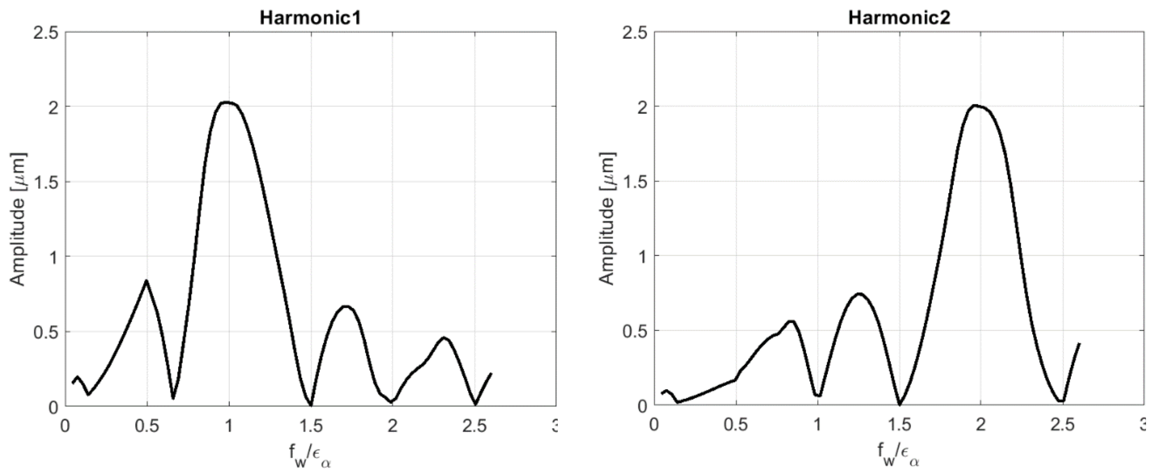
Table 1: Gear mesh data for analysis of profile waviness.

Length Path of Contact	g_{α}	15.931
Transverse Base Pitch	p_{et}	6.910
Base Helix Angle	β_{b}	0



(a) (b) $f_W/\varepsilon_\alpha = 2$

Figure 4: (a) Peak to Peak TE shows maxima for integer values of f_W/ε_α. (b) Amplitude spectrum of TE for $f_W/\varepsilon_\alpha = 2$. Only the 2nd harmonic of the GMF is present.



(a) 1st Harmonic of GMF (b) 2nd Harmonic of GMF

Figure 5: (a) The 1st harmonic of the GMF shows a maximum at $f_W/\varepsilon_\alpha = 1$ and a minimum at $f_W/\varepsilon_\alpha = 2$. (b) The 2nd harmonic of the GMF shows a minimum at $f_W/\varepsilon_\alpha = 1$ and a maximum at $f_W/\varepsilon_\alpha = 2$.

At these values the period of $s(x)$ equals exactly the transverse base pitch p_{et} and all contact points are displaced by the same profile deviation since $s(x) = s(x + p_{et})$. Thus, all theoretical contact points stay in contact and jointly move along each point of the sinusoidal profile topology. Consequently, the PPTE is twice the amplitude.

As can be seen in Figure 5 the first maximum at $f_W/\varepsilon_\alpha = 1$ is reflected only in the first harmonic of the GMF though for $f_\mathrm{W} = \varepsilon_\alpha = 2.3$ there are 3 peaks with excess material along the tooth profile. The second maximum at $f_W/\varepsilon_\alpha = 2$ is reflected only in the 2nd harmonic of the GMF though there are 5 high points along the profile. Our analysis reveals that it is not the number of peaks along the profile of spur gears, but rather the ratio between the distance of the peaks measured along the LOA and the transverse base pitch that determines the frequency content of the TE. If also the profile of the wheel is considered, the same holds for the superimposed profiles. As profile waviness is no design parameter, the goal must be to keep the amplitude as small as possible.

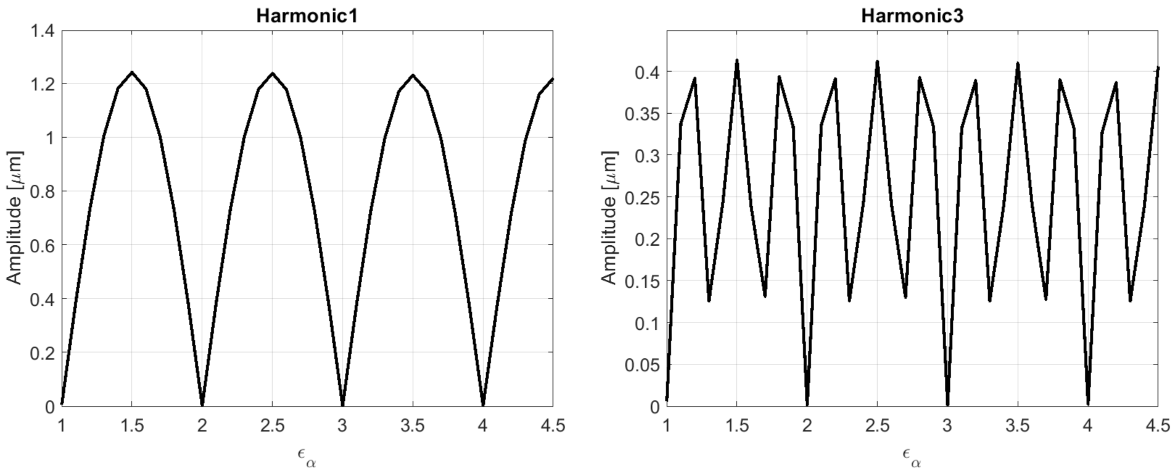
Misconception 2 – TE due to pitch error

The second misconception which we address is that pitch errors produce only sidebands in the amplitude spectrum of the TE. In order to challenge this assumption, we start again from the same spur gear mesh given in Table 1. However, we vary the transverse contact ratio by adapting the transverse base pitch. We approximate the total cumulative pitch deviation F_p over all teeth $i \in \{1,2, \dots, n_T\}$ on the pinion by the sinusoidal function:

$$F_\mathrm{p}(i) = A_\mathrm{p} \sin\left(2\pi \frac{i-0.5}{n_T}\right), \qquad (3)$$

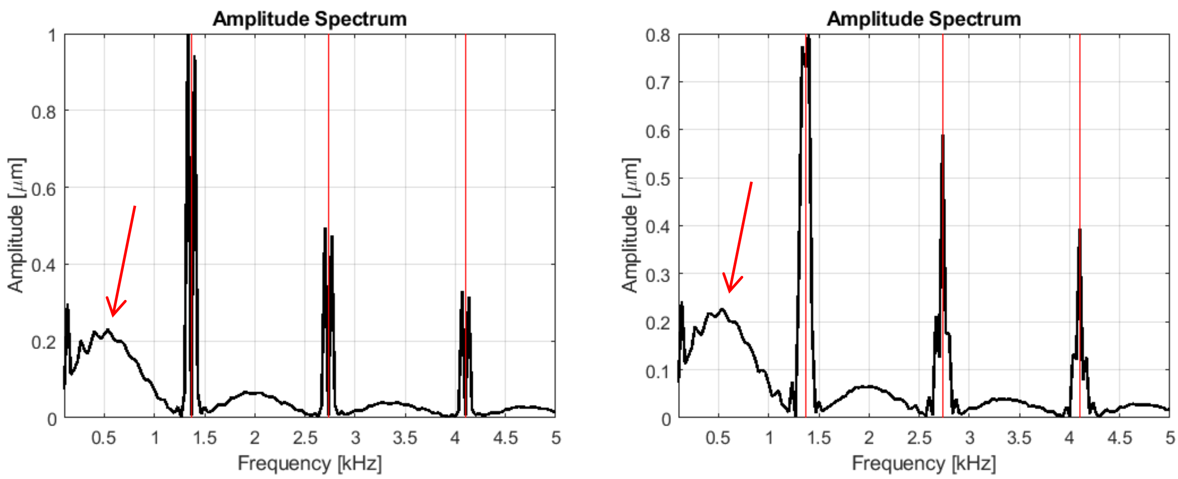
wherein A_p is the amplitude of the pitch error.

Figure 6 shows that the amplitudes of the GMFs are depending on the transverse contact ratio. For integer values of ε_α the TE from pitch deviations has no contribution to any of the GMFs in the amplitude spectrum. For all other values of ε_α, however, the pitch error contributes to the amplitudes of the GMFs. This property of the pitch error was already shown by Kohler and Regan in 1985 [2]. Figure 7 shows the amplitude spectra of the same pitch error F_p for two different transverse contact ratios. Both spectra show high sidebands in the vicinity of the GMFs. While for a contact ratio of $\varepsilon_\alpha = 2$ the amplitudes of the GMFs vanish, for $\varepsilon_\alpha = 1.8$ there are significant contributions to the GMFs. Another interesting phenomenon is the excitation at frequencies well below the first GMF. This is found for all contact ratios and for various forms of $F_\mathrm{p}(i)$. At typical speeds in many operating scenarios associated with moderate loads such as, coasting or city cruising, this low frequency content of the TE is likely to overlap with Eigenfrequencies of the housing. In such situations also the masking noise levels of the wheels and the engine are low. This applies particularly to electric vehicles.



(a) 1st Harmonic of GMF (b) 3rd Harmonic of GMF

Figure 6: (a)+(b): The TE from pitch deviations has no contributions to the harmonics of the GMFs for integer values of the transverse contact ratio.



(a) $\varepsilon_\alpha = 2$ (b) $\varepsilon_\alpha = 1.8$

Figure 7: (a) Zero amplitudes at the GMFs for $\varepsilon_\alpha = 2$. (b) Significant contributions to GMFs for $\varepsilon_\alpha = 1.8$. Remarkable amplitudes are present at frequencies below the 1st GMF (arrow).

4. Susceptibility of TE to misalignment is modulated by bias

Misalignment of the gear mesh may originate from, e.g., shaft and bearing deformation under load, mounting errors, or operating clearance in the bearings. Any misalignment can be interpreted as a deviation from the optimal helix angle in the mesh. Therefore, in order to compensate for mesh misalignment gears are designed to have helix angle modifications. The optimal helix angle modification, however, is load dependent and gears in automotive transmissions are usually optimized to compensate for the misalignment in the higher load range. Also, the target values for microgeometry modifications are often determined by simulations which necessarily comes along with uncertainties in model parameters. So, in reality the actual misalignment may differ significantly.

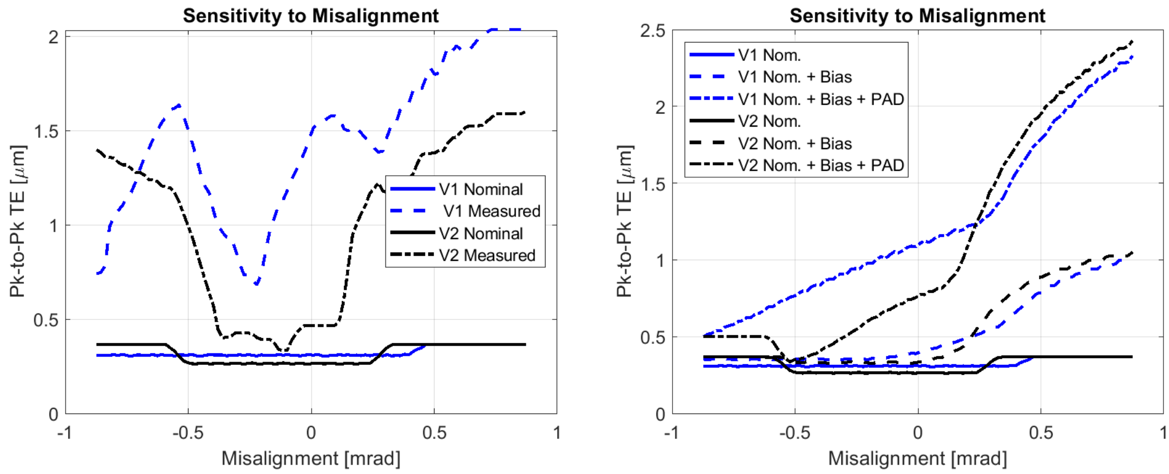
Consider the gear mesh summarized in Table 2. We analyze 2 different microgeometry designs differing in the magnitude of the lead crowning and the helix angle modification as reported in Table 3. Both variants were manufactured in compliance with the specified tolerances. In the simulation, both variants performed equally good with respect to gear whine. In the NVH measurement, however, there were significant differences between both variants. One guess is that the misalignment might be the root cause for the noise. To test this assumption, we imported the measured microgeometries for both variants and varied the misalignment. Figure 8(a) shows that the nominal microgeometries are insensitive to misalignment but the actual topologies show remarkable differences. Investigation of the CMM reports led to the hypothesis that bias and profile angle deviation modulate the sensitivity to misalignment. The results shown in Figure 8(b) confirm that both variants become more susceptible to misalignment if bias and pressure angle deviation is superimposed to the nominal microgeometry. The instantaneous increase of the peak to peak TE for small misalignments is particularly high for Variant V1.

Table 2: Gear mesh data for analysis of sensitivity to misalignment.

Length Path of Contact	g_α	11.39
Transverse Base Pitch	p_{et}	5.750
Base Helix Angle	β_{b}	17.254
Gear Width	b	38.000

Table 3: Differences between microgeometry variants.

	Modification V1	Modification V2
Helix Angle Modification	Large	Small
Lead Crowning	Large	Small



(a) Measured vs. Nominal (b) Nominal + Bias + PAD

Figure 8: (a) The nominal microgeometries are insensitive to misalignment, whereas the measured gears show significant differences. (b) Parameters which modulate the susceptibility to misalignment are bias and pressure angle deviation (PAD).

5. Conclusion

We presented a simple computational model for the investigation of the impact of manufacturing deviations on the TE and its amplitude spectrum for scenarios in which the tooth deflections are small. We addressed two widespread misconceptions. First, we demonstrated that for spur gears with profile waviness that the distance of the peaks in relation to the transverse base pitch determines the frequencies content of the TE. Second, we showed that in addition to high sidebands in the vicinity of the harmonics of the gear mesh frequency, the pitch error excites also the GMFs themselves and enriches the spectrum with considerable amplitudes below the GMF. Finally, we gave an example for how the presented method can be applied in troubleshooting existing NVH issues.

6. References

[1] Stiller, S., Korrektureinfluss Lastverteilung: Programmanleitung zum Korrekturprogramm SEKOR, Forschungsvereinigung Antriebstechnik e.V., FVA-Nr. 609, 2014

[2] Kohler, H. and Regan, R., The derivation of transmission error from pitch error records. Proc. Instn. Mech. Engrs.,199, 1985, pp. 195-201